<u>TITLE OF THE INVENTION</u>

INTERPOLATION METHOD, APPARATUS FOR CARRYING OUT THE

METHOD, AND CONTROL PROGRAM FOR IMPLEMENTING THE METHOD

5

<u>BACKGROUND OF THE INVENTION</u>

Field of the Invention

The present invention relates to an interpolation

10   method that is used in computational fluid dynamics for

realizing the motion of a fluid on a computer, an

apparatus for carrying out the method, and a program

for implementing the method.

15   Description of the Related Art

In recent years, computational fluid dynamics that

realizes the motion of a fluid on a computer has made

remarkable progress. However, it is difficult to

describe the motion of an object in a fluid, or the

20   motion of two fluids or a multi-layer fluid, and

therefore, various approaches have been made to this

difficult theme.

To compute a fluid flow around a stationary object,

a grid can be generated according to the shape of the

25   object, and it is a conventionally known method of the

computation to utilize an unstructured grid or the like

based on a finite element method or the like. When the

object is moved, a method of changing the grid

according to the motion of the object can be employed.

However, if the change of the grid involves large

deformation, it is generally difficult to perform the

5   computation.

On the other hand, a difference equation that

approximately realizes a fluid equation describing a

fluid flow on a structured grid, by using duality

between a grid (primary grid) and a dual grid known as

10   a staggered grid, is highly reliable due to its

stability. This leads to an idea of describing a

volume of a substance in each cell as a function on a

grid, and moving the function. One example of the

method based on this idea utilizes the Volume-of-Fluid

15   method (hereinafter referred to as "the VOF method")

for computing a flow of an incompressible fluid. In

this case, it is a key technique to move a function

describing the volume of the substance of the fluid.

Further, in the case of a compressible fluid, the

20   computation of a flow thereof involves key problems of

defining motions of various objects, and in particular,

propagation of a shock wave and the like are very

closely related with the above-mentioned theme of

describing the motion of a substance or an interface.

25   An initial-value problem concerning an equation

(referred to as "an advection equation") describing

such a motion is called the Riemann problem after a

paper by Riemann on a wave equation concerning a wave in the air "Ueber die Fortpflanzung ebener Luftwellen von endlicher Schwingungsweite (Albhandlungen der Koniglichen Gesellschaft der Wissenschaften zu

5   Gottingen, 8 (1860), 43-65".

As numerical solutions to the Riemann problem, there have been proposed various concepts and schemes, independent of each other or dependent on each other, including a high-order accuracy upwind scheme called

10   the Total Variation Diminishing scheme (hereinafter referred to as "the TVD scheme"), the Essentially Non-Oscillatory scheme (hereinafter referred to as "the ENO scheme") which introduces a limiter function, the Monotone Upstream Centered Scheme for Conservation Laws

15   (hereinafter referred to as "the MUSCL scheme", and the Positivity-Preserving scheme (hereinafter referred to as "the PP scheme".

One of equations dealt with in the present invention is a one-dimensional partial differential

20   equation defined by:

$$\partial F(x,t)/\partial t = \partial C(x,t)F(x,t)/\partial x \quad \ldots(1)$$

To perform computation based on the equation, we will now consider the problem of converting this equation into a difference equation. In the following,

25   arguments X, T of a function F(X,T) represent a discrete grid point in space and a discrete time point, respectively.

To solve the equation (1), there have been proposed various computing methods in the computational fluid dynamics as described above. The TVD scheme is to perform the computation under a TVD condition that a

5   TVD value $TV(F[\cdot,n])$ defined by:

$$TV(F[\cdot,n])=\Sigma j|F[j+1,n]-F[j,n]| \quad …(2)$$

does not increase with time evolution, the TVD condition being defined by the following inequality:

$$TV(F[\cdot,n+1])\leq TV(F[\cdot,n]) \quad …(3)$$

10   The MUSCL scheme, in a broader sense, is a computational method composed of data reconstruction and an approximate Riemann solution.

The ENO scheme is an upwind method with a limiter function in which the MUSCL scheme and the TVD scheme

15   are combined to eliminate vibration occurring when there are discontinuities in the above-described function.

Further, for data reconstruction, there are conventionally employed a method of interpolating the

20   above-described function with a piecewise constant function (which gives first-order accuracy), a method of interpolating the above-described function with a piecewise linear function (which gives second-order accuracy), and a method of interpolating the above-

25   described function by a piecewise parabolic function (which gives third-order accuracy).

The present invention relates to improvement in

the second-order accuracy of interpolation.

Now, a description will be given of a conventional second-order ENO scheme.

First, a description will be given of a method of numerically solving the above equation (1) by finding a solution thereto by finite difference approximation, assuming that there is a piecewise linear function:

$$H[I,J](x)=F(I,J)+G(I,J)x \quad ...(4)$$

If the function $C(I+1/2,J)$ is positive, there is given the following definition:

$$Q(I+1/2,J):=C(I+1/2,J)H[I,J](\Delta x/2-C(I,J)\Delta t/2) \quad ...(5)$$

If the function is not positive, there is given the following definition:

$$Q(I+1/2,J)$$
$$:=C(I+1/2,J)H[I+1,J](-\Delta x/2-C(I+1,J)\Delta t/2) \quad ...(6)$$

The above equation (1) can be approximately solved by obtaining, from the above definitions, the following equation:

$$F(I,J+1)$$
$$=F(I,J)+\Delta t(Q(I+1/2,J)-Q(I-1/2,J))/\Delta x \quad ...(7)$$

On the other hand, a slope function $G(I,J)$ representative of the slope of the piecewise linear function of the equation (4) is defined by the following equation:

$$G(I,J)=\Delta F(I,J)/\Delta x \quad ...(8)$$

Now, there are various methods of defining

$\Delta F(I,J)$, as described hereinafter, and typical three of them will be described here.

First, there are introduced the following definitions:

5        $Sp(I,J):=F(I+1,J)-F(I,J)$

         $Sm(I,J):=F(I,J)-F(I-1,J)$ …(9)

A TVD limiter function in one dimension called the minmod function gives $\Delta F(I,J)$ defined by:

         $\Delta F(I,J)=(sgn(Sp(I,J))+sgn(Sm(I,J)))$

10       $Min(|Sp(I,J)|,|Sm(I,J)|)/2$ …(10)

A TVD limiter function in one dimension called the averaging limiter function gives $\Delta F(I,J)$ defined by:

         $\Delta F(I,J)=(sgn(Sp(I,J))+sgn(Sm(I,J)))$

         $Min(|Sp(I,J)+Sm(I,J)|/2,2|Sp(I,J)|,2|Sm(I,J)|)/2$

15   …(11)

A TVD limiter function in one dimension called the superbee limiter function gives $\Delta F(I,J)$ defined by:

         $\Delta F(I,J)=(sgn(Sp(I,J))+sgn(Sm(I,J)))$

         $Min(Max(|Sp(I,J)|,|Sm(I,J)|)),$

20       $2|Sp(I,J)|,2|Sm(I,J)|)/2$ …(12)

In the above equations (10) to (12), sgn( ) represents a sign function. That is, when the argument thereof is positive i.e. of a plus sign, the function gives a value of 1, while the same is negative i.e. of

25 a minus sign, the function gives a value of -1.

These methods are described e.g. in "A. Suresh, "Positivity-Preserving Schemes in Multidimensions" SIAM

J. Sci. Comput. 22, 1184-1198(2000)".

Using these interpolation methods, a function describing a square shape was subjected to advection calculation using the above equation (7). FIGS. 7 to 9

5   show results of the calculation. More specifically, in FIGS. 7 to 9, shapes are depicted which are given by the function after 100 steps under periodic boundary conditions, using the minmod limiter function, the averaging limiter function, and the superbee limiter

10   function, together with the initial shape for comparison. As is clear from FIGS. 7 to 9, the square shape as the initial shape is drastically lost into dull shapes.

Next, a description will be given of an algorithm

15   for a higher dimension obtained by extending the algorithm for one dimension described heretofore. A method employed in the algorithm for a higher dimension described here is disclosed in the above-mentioned literature.

20   Suppose that $F(I,J,T)$ represents a function of a point on a two-dimensional grid at a given time T, wherein I and J are integers and cooperatively represent a grid point on the two-dimensional grid.

For the function, there are defined the following

25   values:

$Vmin=Min(F(I-1,J+1,T)-F(I,J,T),F(I,J+1,T)-F(I,J,T),$
$F(I+1,J+1,T)-F(I,J,T),F(I-1,J,T)-F(I,J,T),-\varepsilon,$

$$F(I+1,J,T)-F(I,J,T),F(I-1,J-1,T)-F(I,J,T),$$

$$F(I,J-1,T)-F(I,J,T),F(I+1,J-1,T)-F(I,J,T))$$

$$Vmax=Max(F(I-1,J+1,T)-F(I,J,T),F(I,J+1,T)-F(I,J,T),$$

$$F(I+1,J+1,T)-F(I,J,T),F(I-1,J,T)-F(I,J,T),\varepsilon,$$

5
$$F(I+1,J,T)-F(I,J,T),F(I-1,J-1,T)-F(I,J,T),$$

$$F(I,J-1,T)-F(I,J,T),F(I+1,J-1,T)-F(I,J,T))\quad \ldots(13)$$

Further, there are defined:

$$Sx(I,J,T)=(F(I+1,J,T)-F(I-1,J,T))/2$$

$$Sy(I,J,T)=(F(I,J+1,T)-F(I,J-1,T))/2\quad \ldots(14)$$

10
$$V=2(Min(|Vmin|,|Vmax|))/(|Sx(I,J,T)|$$

$$+|Sx(I,J,T)|+\varepsilon)\quad \ldots(15)$$

At this time, after defining:

$$Gx(I,J,T)=Min(1,V)Sx(I,J,T)/\Delta x$$

$$Gy(I,J,T)=Min(1,V)Sy(I,J,T)/\Delta y\quad \ldots(16)$$

15　an interpolation function $H(I,J,T)(x,y)$ for a grid

point (I,J) is defined as:

$$H(I,J,T)(x,y)=F(I,J,T)+Gx(I,J,T)x+Gy(I,J,T)y\quad \ldots(17)$$

By using the method, with respect to initial

values of a function shown in FIG. 10A, advection

20　calculation of the function can be carried, a result of

which is shown in FIG. 10B.

However, the conventional interpolation method

suffers from the following problem:

In the Volume-of-Fluid (VOF) method applied to a

25　two-phase incompressible fluid, how to cause time

evolution of a shape-describing function descriptive of

a shape as a result of the advection calculation while

preserving the shape described by the function is a matter of great interest, but the conventional interpolation method tends to cause progressive loss of the initial shape into a dull shape, with execution of

5   computational steps.

In view of this problem, a computational algorithm enabling the shape to be preserved more effectively is indispensable, and to this end, for an algorithm for one dimension, it is necessary to replace the

10   interpolation function of the equation (4) by an improved one, and for an algorithm for a higher dimension, it is necessary to replace the interpolation function of the equation (17) by an improved one.

15   SUMMARY OF THE INVENTION

The present invention has been made in view of the above problems of the prior art, and it is an object of the present invention is to provide an interpolation

20   method for the Volume-of-Fluid (VOF) method applied to a two-phase incompressible fluid, in particular, which makes it possible to cause time evolution of a shape-describing function based on the Volume-of-Fluid (VOF) method while preserving a sharpness of a shape

25   described by the function, an apparatus for carrying out the method, and a control program for implementing the method.

To attain the above object, in a first aspect of the present invention, there is provided an interpolation method of defining a function F on a one-dimensional structured grid formed on a one-dimensional

5 real region, the function being defined through definition of a value thereof at a center of each cell within the one-dimensional structured grid, as an interpolation function H, the method comprising the steps of setting, with respect to a cell of interest on

10 the one-dimensional structured grid, a slope to zero if a forward difference and a backward difference of the function f have different signs, and to a value twice as large as a smaller one of absolute values of the forward difference and the backward difference if the

15 forward difference and the backward difference have the same sign, and defining the function F on a partial region of the one-dimensional real region determined by the cell of interest, by a linear function having a value of F0 at a center of the cell of interest and the

20 slope.

With the arrangement of the interpolation method according to the first aspect of the present invention, it is possible to realize an interpolation algorithm for the Volume-of-Fluid (VOF) method applied to a two-

25 phase incompressible fluid, in particular, which makes it possible to cause time evolution of a shape-describing function based on the Volume-of-Fluid (VOF)

method while preserving a sharpness of a shape described by the function, and the function gives very excellent results of describing the shape.

Preferably, the interpolation method is applied to
5 as a numerical solution of an advection-type differential equation.

To attain the above object, in a second aspect of the present invention, there is provided an interpolation method of defining a function F on a two-
10 dimensional structured grid formed on a two-dimensional real region, the function being defined through definition of a value thereof at a center of each cell within the two-dimensional structured grid, as an interpolation function H, the method comprising the
15 steps of setting a cell of interest to a cell A, the cell A having a first side extending in an x direction, a second side extending in the x direction and being opposite to the first side, a third side extending in a y direction, and a fourth side extending in the y
20 direction and being opposite to the third side, defining values twice as large as values of one-sided differences of the function F between a center of the cell A and respective centers of four cells adjacent to the cell A on the first side, the second side, the
25 third side, and the fourth side, as a first-sided difference (DFxmin), a second-sided difference (DFxmax), a third-sided difference (DFymin), and a fourth-sided

difference (DFymax), respectively, and setting an x-
direction difference to zero if the first-sided
difference and the second-sided difference have
different signs, and to a smaller one of absolute

5   values of the first-sided difference and the second-
sided difference if the first-sided difference and the
second-sided difference have the same sign, and a y-
direction difference to zero if the third-sided
difference and the fourth-sided difference have

10   different signs, and to a smaller one of absolute
values of the third-sided difference and the fourth-
sided difference if the third-sided differences and the
fourth-sided difference have the same sign, forming an
interpolation function candidate which has slopes

15   defined by the x-direction difference and the y-
direction difference, respectively, and has a value $F0$
of the function F at the center of the cell A and
setting this candidate to a plane B, modifying, if a
value of the plane B at a first grid point at a

20   location of intersection of the first side and the
third side of the cell A is larger than a value of the
center of the cell A, the plane B by multiplying the x-
direction difference and the y-direction difference by
a largest constant not more than 1 such that the value

25   of the plane B at the first grid point does not exceed
any of values of the function F at respective centers
of three cells having the first grid point in common

except for the cell A, modifying, if the value of the plane B at the first grid point is smaller than the value of the center of the cell A, the plane B by multiplying the x-direction difference and the y-

5   direction difference by a largest constant not more than 1 such that the value of the plane B at the first grid point does not fall below any of the values of the function F at the respective centers of the three cells having the grid point in common except for the cell A,

10   and carrying out, on the plane B thus obtained, the same operation as carried out as to the first grid point, as to a second grid point at a location of intersection of the first side and the fourth side of the cell A, a third grid point at a location of

15   intersection of the second side and the third side, and a fourth grid point at a location of intersection of the second side and the fourth side, to thereby change the slope of the plane B, and define the resulting plane as the interpolation function within the cell A.

20   With the arrangement of the interpolation method according to the second aspect of the present invention, it is possible to obtain the same advantageous effects as provided by the first aspect of the present invention.

25   To attain the above object, in a third aspect of the present invention, there is provided an interpolation method of defining a function F on a

three-dimensional structured grid formed on a three-dimensional real region, the function being defined through definition of a value thereof at a center of each cell within the three-dimensional structured grid,

5 as an interpolation function H, the method comprising the steps of setting a cell of interest to a cell A, the cell A having a first side extending in an x direction, a second side extending in the x direction and being opposite to the first side, a third side

10 extending in a y direction, a fourth side extending in the y direction and being opposite to the third side, a fifth side in a z direction, and a sixth side extending in the z direction and being opposite to the fifth side, defining values twice as large as values of one-sided

15 differences of the function F between a center of the cell A and respective centers of six cells adjacent to the cell A on the first side, the second side, the third side, the fourth side, the fifth side, and the sixth side, as a first-sided difference (DFxmin), a

20 second-sided difference (DFxmax), a third-sided difference (DFymin), a fourth-sided difference (DFymax), a fifth-sided difference (DFzmax), and a sixth-sided difference (DFzmin), respectively, and setting an x-direction difference to zero if the first-sided

25 difference and the second-sided difference have different signs, and to a smaller one of absolute values of the first-sided difference and the second-

sided difference if the first-sided difference and the
second-sided difference have the same sign, a y-
direction difference to zero if the third-sided

5   difference and the fourth-sided difference have
different signs, and to a smaller one of absolute
values of the third-sided difference and the fourth-
sided difference if the third-sided differences and the
fourth-sided difference have the same sign, and a z-

10   direction difference to zero if the fifth-sided
difference and the sixth-sided difference have
different signs, and to a smaller one of absolute
values of the fifth-sided difference and the sixth-
sided difference if the fifth-sided difference have

15   the same sign, forming an
interpolation function candidate which has slopes
defined by the x-direction difference, the y-direction
difference, and the z-direction difference,
respectively, and has a value F0 of the function F at

20   the center of the cell A and setting this candidate to
a plane B, modifying, if a value of the plane B at a
first grid point at a location of intersection of the
first side, the third side, and the fifth side of the
cell A is larger than the value of the center of the

25   cell A, the plane B by multiplying the x-direction
difference, the y-direction difference, and the z-
direction difference, by a largest constant not more
than 1 such that the value of the plane B at the first

grid point does not exceed any of the values of the
function F at respective centers of seven cells having
the first grid point in common except for the cell A,
modifying, if the value of the plane B at the first

5   grid point is smaller than the value of the center of
the cell A, the plane B by multiplying the x-direction
difference, the y-direction difference, and the z-
direction difference, by a largest constant not more
than 1 such that the value of the plane B at the first

10   grid point does not fall below any of the values of the
function F at the respective centers of the seven cells
having the grid point in common except for the cell A,
and carrying out, on the plane B thus obtained, the
same operation as carried out as to the first grid

15   point, on a second grid point at a location of
intersection of the first side, the third side, and the
sixth side, a third grid point at a location of
intersection of the first side, the fourth side, and
the fifth side, a fourth grid point at a location of

20   intersection of the first side, the fourth side, and
the sixth side, a fifth grid point at a location of
intersection of the second side, the third side, and
the fifth side, and a sixth grid point at a location of
intersection of the second side, the third side, and

25   the sixth side, a seventh grid point at a location of
intersection of the second side, the fourth side, and
the fifth side, and an eighth grid point at a location

of intersection of the second side, the fourth side, and the sixth side, to thereby change the slope of the plane, and define the resulting plane as the interpolation function within the cell A.

5   With the arrangement of the interpolation method according to the third aspect of the present invention, it is possible to obtain the same advantageous effects as provided by the first aspect of the present invention.

10   To attain the above object, in a fourth aspect of the present invention, there is provided an apparatus for carrying out an interpolation method of defining a function F on a one-dimensional structured grid formed on a one-dimensional real region, the function being

15   defined through definition of a value thereof at a center of each cell within the one-dimensional structured grid, as an interpolation function H, the apparatus comprising a setting device that sets, with respect to a cell of interest on the one-dimensional

20   structured grid, a slope to zero if a forward difference and a backward difference of the function F have different signs, and to a value twice as large as a smaller one of absolute values of the forward difference and the backward difference, and a

25   definition device that defines the function F on a partial region of the one-dimensional real region determined by the cell of interest, by a linear

function having a value of F0 at a center of the cell of interest and the slope.

With the arrangement of the apparatus according to the fourth aspect of the present invention, it is
5   possible to obtain the same advantageous effects as provided by the first aspect of the present invention.

To attain the above object, in a fifth aspect of the present invention, there is provided an apparatus for carrying out an interpolation method of defining a
10   function F on a two-dimensional structured grid formed on a two-dimensional real region, the function being defined through definition of a value thereof at a center of each cell within the two-dimensional structured grid, as an interpolation function H, the
15   apparatus comprising a cell-setting device that sets a cell of interest to a cell A, the cell A having a first side extending in an x direction, a second side extending in the x direction and being opposite to the first side, a third side extending in a y direction,
20   and a fourth side extending in the y direction and being opposite to the third side, a difference-setting device that defines values twice as large as values of one-sided differences of the function F between a center of the cell A and respective centers of four
25   cells adjacent to the cell A on the first side, the second side, the third side, and the fourth side, as a first-sided difference (DFxmin), a second-sided

difference (DFxmax), a third-sided difference (DFymin),

and a fourth-sided difference (DFymax), respectively,

and sets an x-direction difference to zero if the

first-sided difference and the second-sided difference

5   have different signs, and to a smaller one of absolute

values of the first-sided difference and the second-

sided difference if the first-sided difference and the

second-sided difference have the same sign, and a y-

direction difference to zero if the third-sided

10   difference and the fourth-sided difference have

different signs, and to a smaller one of absolute

values of the third-sided difference and the fourth-

sided difference if the third-sided differences and the

fourth-sided difference have the same sign, an

15   interpolation function candidate-forming device that

forms an interpolation function candidate which has

slopes defined by the x-direction difference and the y-

direction difference, respectively, and has a value F0

of the function F at the center of the cell A and

20   setting this candidate to a plane B, a first

modification device that modifies, if a value of the

plane B at a first grid point at a location of

intersection of the first side and the third side of

the cell A is larger than a value of the center of the

25   cell A, the plane B by multiplying the x-direction

difference and the y-direction difference by a largest

constant not more than 1 such that the value of the

plane B at the first grid point does not exceed any of

values of the function F at respective centers of three

cells having the first grid point in common except for

the cell A, a second modification device that modifies,

5   if the value of the plane B at the first grid point is

smaller than the value of the center of the cell A, the

plane B by multiplying the x-direction difference and

the y-direction difference by a largest constant not

more than 1 such that the value of the plane B at the

10   first grid point does not fall below any of the values

of the function F at the respective centers of the

three cells having the grid point in common except for

the cell A, and a definition device that carries out,

on the plane B thus obtained, the same operation as

15   carried out as to the first grid point, as to a second

grid point at a location of intersection of the first

side and the fourth side of the cell A, a third grid

point at a location of intersection of the second side

and the third side, and a fourth grid point at a

20   location of intersection of the second side and the

fourth side, to thereby change the slope of the plane B,

and define the resulting plane as the interpolation

function within the cell A.

    With the arrangement of the apparatus according to

25   the fifth aspect of the present invention, it is

possible to obtain the same advantageous effects as

provided by the first aspect of the present invention.

To attain the above object, in a sixth aspect of the present invention, there is provided an apparatus for carrying out an interpolation method of defining a function F on a three-dimensional structured grid

5 formed on a three-dimensional real region, the function being defined through definition of a value thereof at a center of each cell within the three-dimensional structured grid, as an interpolation function H, the apparatus comprising a cell-setting device that sets a

10 cell of interest to a cell A, the cell A having a first side extending in an x direction, a second side extending in the x direction and being opposite to the first side, a third side extending in a y direction, a fourth side extending in the y direction and being

15 opposite to the third side, a fifth side in a z direction, and a sixth side extending in the z direction and being opposite to the fifth side, a difference-setting device that defines values twice as large as values of one-sided differences of the

20 function F between a center of the cell A and respective centers of six cells adjacent to the cell A on the first side, the second side, the third side, the fourth side, the fifth side, and the sixth side, as a first-sided difference (DFxmin), a second-sided

25 difference (DFxmax), a third-sided difference (DFymin), a fourth-sided difference (DFymax), a fifth-sided difference (DFzmax), and a sixth-sided difference

(DFzmin), respectively, and sets an x-direction
difference to zero if the first-sided difference and
the second-sided difference have different signs, and
to a smaller one of absolute values of the first-sided

5   difference and the second-sided difference if the
first-sided difference and the second-sided difference
have the same sign, a y-direction difference to zero if
the third-sided difference and the fourth-sided
difference have different signs, and to a smaller one

10   of absolute values of the third-sided difference and
the fourth-sided difference if the third-sided
differences and the fourth-sided difference have the
same sign, and a z-direction difference to zero if the
fifth-sided difference and the sixth-sided difference

15   have different signs, and to a smaller one of absolute
values of the fifth-sided difference and the sixth-
sided difference if the fifth-sided difference and the
sixth-sided difference have the same sign, an
interpolation function candidate-forming device that

20   forms an interpolation function candidate which has
slopes defined by the x-direction difference, the y-
direction difference, and the z-direction difference,
respectively, and has a value $F0$ of the function F at
the center of the cell A, and setting this candidate to

25   a plane B, a first modification device that modifies,
if a value of the plane B at a first grid point at a
location of intersection of the first side, the third

side, and the fifth side of the cell A is larger than

the value of the center of the cell A, the plane B by

multiplying the x-direction difference, the y-direction

difference, and the z-direction difference, by a

5   largest constant not more than 1 such that the value of

the plane B at the first grid point does not exceed any

of the values of the function F at respective centers

of seven cells having the first grid point in common

except for the cell A, a second modification device

10   that modifies, if the value of the plane B at the first

grid point is smaller than the value of the center of

the cell A, the plane B by multiplying the x-direction

difference, the y-direction difference, and the z-

direction difference, by a largest constant not more

15   than 1 such that the value of the plane B at the first

grid point does not fall below any of the values of the

function F at the respective centers of the seven cells

having the grid point in common except for the cell A,

and a definition device that carries out, on the plane

20   B thus obtained, the same operation as carried out as

to the first grid point, on a second grid point at a

location of intersection of the first side, the third

side, and the sixth side, a third grid point at a

location of intersection of the first side, the fourth

25   side, and the fifth side, a fourth grid point at a

location of intersection of the first side, the fourth

side, and the sixth side, a fifth grid point at a

location of intersection of the second side, the third side, and the fifth side, and a sixth grid point at a location of intersection of the second side, the third side, and the sixth side, a seventh grid point at a

5 location of intersection of the second side, the fourth side, and the fifth side, and an eighth grid point at a location of intersection of the second side, the fourth side, and the sixth side, to thereby change the slope of the plane, and define the resulting plane as the

10 interpolation function within the cell A.

With the arrangement of the apparatus according to the sixth aspect of the present invention, it is possible to obtain the same advantageous effects as provided by the first aspect of the present invention.

15 To attain the above object, in a seventh aspect of the present invention, there is provided a control program for causing a computer to execute the interpolation method as recited hereinabove as the interpolation method according to the first aspect of

20 the present invention.

To attain the above object, in an eighth aspect of the present invention, there is provided a control program for causing a computer to execute the interpolation method as recited hereinabove as the

25 interpolation method according to the second aspect of the present invention.

To attain the above object, in a ninth aspect of

the present invention, there is provided a control

program for causing a computer to execute the

interpolation method as recited hereinabove as the

interpolation method according to the third aspect of

5   the present invention.

With the arrangement of each of the control

programs according to the seventh to ninth aspects of

the present invention, it is possible to obtain the

same advantageous effects as provided by the first

10   aspect of the present invention.

The above and other objects, features, and

advantages of the invention will become more apparent

from the following detailed description taken in

conjunction with the accompanying drawings.

15

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a flowchart showing a process of

advection calculation using an interpolation method

20   according to a first embodiment of the present

invention;

FIG. 2 is a diagram showing results of the

advection calculation using the interpolation method

illustrated in FIG. 1;

25   FIG. 3 is a perspective view of a computer system

for executing the advection calculation using the

interpolation method illustrated in FIG. 1;

FIG. 4 is a block diagram showing the arrangement of essential components of the computer system using the interpolation method illustrated in FIG. 1;

FIG. 5 is a flowchart showing a process of advection calculation using an interpolation method according to a second embodiment of the present invention;

FIGS. 6A and 6B are diagrams showing results of the advection calculation using the interpolation method according to the second embodiment and an interpolation method according to a third embodiment of the present invention;

FIG. 7 is a diagram showing results of advection calculation using a conventional minmod limiter function;

FIG. 8 is a diagram showing results of advection calculation using a conventional averaging limiter function;

FIG. 9 is a diagram showing results of advection calculation using a conventional superbee limiter function; and

FIGS. 10A and 10b are diagrams showing results of advection calculation using a conventional interpolation method (for a higher dimension).

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

The present invention will now be described in detail below with reference to the accompanying drawings showing preferred embodiments thereof. In the drawings, elements and parts which are identical

5 throughout the views are designated by identical reference numeral, and duplicate description thereof is omitted.

First, an interpolation method according to a first embodiment of the invention will be described.

10 The method according to the present embodiment interpolates a function by a piecewise linear function, for data reconstruction. That is, as a numerical solution to an advection equation, the present interpolation method corresponds to a solution of the

15 second-order accuracy.

In the present embodiment, as a problem for one dimension, we will consider the problem of converting a one-dimensional partial differential equation defined by:

20 $\partial F(x,t)/\partial t = \partial C(x,t)F(x,t)/\partial x$ ...(18)

into a difference equation to perform computation based on the equation. In the following, arguments X, T of a function F(X,T) represent a discrete grid point in space and a discrete time point, respectively.

25 First, it is assumed that the interpolation function H[I,J](x) according to the present embodiment has been already obtained as a piecewise linear

function as follows:

$$H[I,J](x)=F(I,J)+G(I,J)x \quad …(19)$$

With this assumption, a description will be given of a method of numerically solving the above equation

5   (18) by finding a solution thereto by finite difference approximation. A method of determining the coefficient $G(I,J)$ of the interpolation function $H[I,J](x)$ of the present embodiment will be described hereinafter. As described later, the method of determining the

10   coefficient $G(I,J)$ characterizes the interpolation algorithm according to the present embodiment. It is essential to the present embodiment that the coefficient $G(I,J)$ is determined by using neighboring information of geometric properties of the grid.

15   In short, when there are provided a one-dimensional structured grid formed on a one-dimensional real region, and a function $F(X,T)$ on the one-dimensional structured grid, the function being defined through definition of a value thereof at the center of

20   each cell within the one-dimensional structured grid, the function $F(X,T)$ is defined as an interpolation function $H[I,J](x)$ on the one-dimensional real region by using the function $F(X,T)$ and the geometry of the grid. In other words, while the function F defines

25   mapping by discrete points alone, the function H extends the definition range of mapping over the entire X axis.

If the function C(I+1/2,J) is positive, there is given the following definition:

$$Q(I+1/2,J):=C(I+1/2,J)H[I,J](\Delta x/2-C(I,J)\Delta t/2)$$

...(20)

5   If the same is not positive, there is given the following definition:

$$Q(I+1/2,J):=C(I+1/2,J)H[I+1,J](-\Delta x/2-C(I,J)\Delta t/2)$$

...(21)

From the above definitions, there is obtained the

10   following equation:

$$F(I,J+1)=F(I,J)+\Delta t(Q(I+1/2,J)-Q(I-1/2,J))/\Delta x$$

...(22)

On the other hand, the slope function G(I,J) representative of the slope of the piecewise linear

15   function is defined in the following manner:

First, there are introduced the following definitions:

$$Sp(I,J):=F(I+1,J)-F(I,J)$$

$$Sm(I,J):=F(I,J)-F(I-1,J) \quad ...(23)$$

20   Further, there is introduced a limiter function in one dimension for the present embodiment, defined by:

$$\Delta F(I,J)=(sgn(Sp(I,J))+sgn(Sm(I,J)))$$

$$Min(|Sp(I,J)|,|Sm(I,J)|) \quad ...(24)$$

In this equation, sgn( ) represents a sign

25   function. That is, when the argument thereof is positive or of a plus sign, the function gives a value of 1, while the same is negative i.e. of a minus, the

function gives a value of -1. It is essential that the value is 2 times larger than the conventional minmod limiter function. By using the $\Delta F(I,J)$, it is only required to define the slope function as:

5        $G(I,J) = \Delta F(I,J)/\Delta x$ ...(25)

An expression $Sp(I,J)/\Delta x$ means a forward difference from the corresponding one of the definitions of the above equations (23), while an expression $Sm(I,J)/\Delta x$ means a backward difference from

10   the corresponding one of the definitions of the same. If the terms $Sp(I,J)$ and $Sm(I,J)$ have different signs, the sum $(sgn(SpI,J) + sgn(Sm(I,J))$ becomes equal to zero, and hence the whole limiter function $\Delta F(I,J)$ and therefore the slope function $G(I,J)$ becomes equal to

15   zero. If the terms do not have different signs, the slope function $G(I,J)$ becomes equal to a value twice as large as the smaller one of the absolute values of the forward difference and the backward difference, since the sum $(sgn(SpI,J) + sgn(Sm(I,J))$ becomes equal to 2

20   or -2.

As described above, according to the interpolation method of the present embodiment, with respect to a cell of interest on the one-dimensional structured grid, the slope of the function $H(I,J)$ is set to zero if the

25   signs of the forward difference and the backward difference of the mapping F on the cell are different from each other, and to a value twice as large as the

smaller one of the absolute values of the differences
if the signs are the same, the value $F(X,T)$ at the
center of the cell of interest is determined as the
linear function, and based on the linear function thus

5 determined, values of the function $H(I,J)$ on a partial
region of the one-dimensional real region determined by
the cell are defined.

  FIG. 1 is a flowchart showing a process of
advection calculation using the above described

10 interpolation method according to the first embodiment.

  First, in a step S1, initialization is performed,
e.g. by generating a grid, determining time step
intervals, and then initializing the initial shape of a
function $F(\cdot,1)$.

15 In the following step S2, the computation of the
above-described equation $Q(\cdot,J)$ (see the equations (20)
and (21)) is performed. At this time, as shown in the
above equations (20) and (21), the interpolation
function H according to the present embodiment is used.

20 Then, in a step S3, based on data obtained by the
equation $Q(\cdot,J)$, the computation of the function
$F(\cdot,J+1)$ is carried out (see the above equation (22)).

  Thereafter, time is updated in a step S4. More
specifically, the problem being solved now is a time

25 evolution equation expressed by the above equation (18),
and according to the conventional practice of numerical
solution of an ordinary time evolution equation, the

value $F(\cdot,J)$ of the function F at time $T=\Delta tJ$ is determined from the value $F(\cdot,J-1)$, and then time is changed from $T=\Delta t*(J-1)$ to $T=\Delta t*J$.

Then, in a step S5, it is determined whether or

5 not the present step is a final step, and if the present step is not a final step, the process returns to the step S2, whereby the present computation is performed until it is determined that the present step is a final step.

10 The use of the interpolation method according to the present embodiment provides two separate regions, one defined by a function with a slope of zero, and the other defined by a function with a sharp slope, which makes it possible to obtain very good results as a

15 function describing the shape of a two-phase incompressible fluid. FIG. 2 shows a result of the computation. The present computation is carried out under the same conditions corresponding to those of the conventional computations results of which are shown in

20 FIGS. 7 to 9, and the shape corresponding to the initial value and that after 1000 steps are depicted in FIG. 2. It will be understood that the result of advection calculation carried out by the interpolation method according to the present embodiment shown in FIG.

25 2 maintains a very sharp rectangular shape compared with those shown in FIGS. 7 to 9.

Next, a description will be given of an apparatus

for carrying out the interpolation method according to the present embodiment.

FIG. 3 is a perspective view of a computer system for executing advection calculation using the interpolation method shown in FIG. 1, and FIG. 4 is a block diagram showing the arrangement of essential components of the computer system.

As shown in FIGS. 3 and 4, the computer system for carrying out the interpolation method is comprised of a processor including a plurality of CPU's 116 and a memory 118, a storage device including a hard disk 117 and a floppy (registered trademark) disk 110, an input section including a keyboard 115 and a mouse 114, and an output display section including a display 112. The processor and other components are accommodated in a main body 111.

This computer system loads a program designed according to the flowchart shown in FIG. 1 and encoded, into the memory 118, reserves storage for necessary computation, performs predetermined arithmetic operations based on a shape of a region, coordinates of grid points, and physical quantities, which have been input by an appropriate method, and writes values of the physical quantities obtained on the region by the arithmetic operations into the hard disk 117 or the like for storage and/or causes the same to be displayed on the display 112.

The use of the interpolation method according to
the present embodiment makes it possible to form a
function on the real axis from values given on the grid
points though the values include discontinuities.

5   Therefore, even when the present invention is applied
to a usage different from the present embodiment, e.g.
data extraction in signal processing, or one-
dimensional image processing, it is possible to express
a sharp shape.

10   Next, a description will be given of an
interpolation method according to a second embodiment
of the present invention. In the second embodiment,
the interpolation algorithm for a one-dimensional grid,
described hereinabove as to the first embodiment, is

15   extended to one for a two-dimensional grid.

The equation dealt with in the present embodiment
is a partial differential equation in a two-dimensional
space, defined by:

$$\partial F(x,y,t)/\partial t = \partial Cx(x,y,t)F(x,y,t)/\partial x$$

20          $$+ \partial Cy(x,y,t)F(x,y,t)/\partial y \quad ...(26)$$

The problem of converting this function to a
difference equation for computation will now be
considered. Suppose a function $F(X,Y,T)$ as a
discretized form of $F(x,y,t)$. The arguments X, Y, in

25   the function F cooperatively represent a discrete
spacial grid point and the argument T in the same
represents a discrete time.

First, it is assumed that an interpolation
function H[I,J,T](x,y) according to the present
embodiment has already been obtained as the following
piecewise linear function:

5 $$H[I,J,T](x,y)=F(I,J,T)+Gx(I,J,T)x+Gy(I,J,T)y \quad ...(27)$$

With this assumption, a description will be given
of a method of numerically solving the above equation
(26) by finding a solution thereto by finite difference
approximation. A method of determining coefficients Gx

10 (I,J,T) and Gy(I,J,T) of the interpolation function
H[I,J,T](x,y) of the present embodiment will be
described hereinafter. As described later, the method
of determining the coefficients Gx(I,J,T) and Gy(I,J,T)
characterizes the interpolation algorithm according to

15 the present embodiment. It is essential to the present
embodiment that the coefficients Gx(I,J,T) and
Gy(I,J,T) are determined by using neighboring
information of geometric properties of the grid.

In short, when there are provided a two-
20 dimensional structured grid formed on a two-dimensional
real region, and a function F(X,Y,T) on the two-
dimensional structured grid, the function being defined
through definition of a value thereof at the center of
each cell within the two-dimensional structured grid,

25 the function F(X,Y,T) is defined as an interpolation
function H[I,J,T](x,y) on the two-dimensional real
region by using the function F(X,Y,T) and the geometry

of the grid.

If the function Cx(I+1/2,J,T) is positive, there is given the following definition:

Qx(I+1/2,J,T):=Cx(I+1/2,J,T)H[I,J,T]

$(\Delta x/2-Cx(I,J,T)\Delta t/2,-Cy(I,J,T)\Delta t/2)$ ...(28)

If the function Cx(I+1/2,J,T) is not positive, there is given the following definition:

Qx(I+1/2,J,T):=Cx(I+1/2,J,T)H[I+1,J,T]

$(-\Delta x/2+Cx(I,J,T)\Delta t/2,-Cy(I,J,T)\Delta t/2)$ ...(29)

If the function Cy(I,J+1/2,T) is positive, there is given the following definition:

Qy(I,J+1/2,J,T):=Cy(I,J+1/2,T)H[I,J,T]

$(-Cx(I,J,T)\Delta t/2,\Delta y/2-Cy(I,J,T)\Delta t/2)$ ...(30)

If the function Cy(I,J+1/2,T) is not positive, there is given the following definition:

Qy(I,J+1/2,J,T):=Cy(I,J+1/2,T)H[I,J+1,T]

$(-Cx(I,J,T)\Delta t/2,-\Delta y/2-Cy(I,J,T)\Delta t/2)$ ...(31)

From the above definitions, there is obtained the following equation:

F(I,J,T+1)=F(I,J,T)+Δt(Qx(I+1/2,J,T)

-Qx(I-1/2,J,T))/Δx+Δt(Qy(I,J+1/2,T)

-Qy(I,J-1/2,T))/Δy ...(32)

On the other hand, a slope function G(I,J,T) representative of the slope of the piecewise linear function is defined in the following manner:

First, there are introduced the following definitions:

$$Sxp(I,J,T):=F(I+1,J,T)-F(I,J,T)$$

$$Sxm(I,J,T):=F(I,J,T)-F(I-1,J,T)$$

$$Syp(I,J,T):=F(I,J+1,T)-F(I,J,T)$$

$$Sym(I,J,T):=F(I,J,T)-F(I,J-1,T) \quad ...(33)$$

Further, there are introduced limiter functions in one dimension for the present embodiment, defined by:

$$Sx(I,J)=(sgn(Sxp(I,J,T))+sgn(Sxm(I,J,T)))$$

$$Min(|Sxp(I,J,T)|,|Sxm(I,J,T)|)$$

$$Sy(I,J)=(sgn(Syp(I,J,T))+sgn(Sym(I,J,T)))$$

$$Min(|Syp(I,J,T)|,|Sym(I,J,T)|) \quad ...(34)$$

Then, there are provided the following definitions:

$$Vppmin=Min(F(I,J+1,T)-F(I,J,T),F(I+1,J+1,T)$$
$$-F(I,J,T),-\varepsilon,F(I+1,J,T)-F(I,J,T))$$

$$Vmpmin=Min(F(I-1,J+1,T)-F(I,J,T),F(I,J+1,T)$$
$$-F(I,J,T),F(I-1,J,T)-F(I,J,T),-\varepsilon)$$

$$Vpmmin=Min(-\varepsilon,F(I+1,J,T)-F(I,J,T),F(I,J-1,T)$$
$$-F(I,J,T),F(I+1,J-1,T)-F(I,J,T))$$

$$Vmmmin=Min(F(I-1,J,T)-F(I,J,T),-\varepsilon,F(I-1,J-1,T)$$
$$-F(I,J,T),F(I,J-1,T)-F(I,J,T)) \quad ...(35)$$

$$Vppmax=Max(F(I,J+1,T)-F(I,J,T),F(I+1,J+1,T)$$
$$-F(I,J,T),-\varepsilon,F(I+1,J,T)-F(I,J,T))$$

$$Vmpmax=Max(F(I-1,J+1,T)-F(I,J,T),F(I,J+1,T)$$
$$-F(I,J,T),F(I-1,J,T)-F(I,J,T),-\varepsilon)$$

$$Vpmmax=Max(-\varepsilon,F(I+1,J,T)-F(I,J,T),F(I,J-1,T)$$
$$-F(I,J,T),F(I+1,J-1,T)-F(I,J,T))$$

$$Vmmmax=Max(F(I-1,J,T)-F(I,J,T),-\varepsilon,F(I-1,J-1,T)$$

-F(I,J,T),F(I,J-1,T)-F(I,J,T)) ...(36)

There are also provided the following definitions:

hpp(I,J,T)=Sx(I,J,T)/2+Sx(I,J,T)/2

hmp(I,J,T)=-Sx(I,J,T)/2+Sx(I,J,T)/2

5 hpm(I,J,T)=Sx(I,J,T)/2-Sx(I,J,T)/2

hmm(I,J,T)=-Sx(I,J,T)/2-Sx(I,J,T)/2 ...(37)

Further, there is performed the following calculation:

Assuming, for the time being, that an equation

10 phipp=phimp=phipm=phimm=1 holds, if hpp>Vppmax holds, an equation of phipp=Vppmax/hpp is newly defined, while if hpp<Vppmin holds, an equation of phipp=Vppmin/hpp is newly defined;

Similarly, if hpm>Vpmmax holds, an equation of

15 phipm=Vpmmax/hpm is newly defined, while if hpm<Vpmmin holds, an equation of phipm=Vpmmin/hpm is newly defined;

Also, if hmp>Vmpmax holds, an equation of phimp=Vmpmax/hmp is newly defined, while if hmp<Vmpmin

20 holds, an equation of phimp=Vmpmin/hmp is newly defined; and

If hmm>Vmmmax holds, an equation of phimm=Vmmmax/hmm is newly defined, while if hmm<Vmmmin holds, an equation of phimm=Vmmmin/hmm is newly defined.

25 By using the thus obtained values phipp, phimp, phipm, phimm, the above-mentioned coefficients Gx(I,J,T) and Gy(I,J,T) are determined by the following

equations:

Gx(I,J,T)

=Min(phipp,phipm,phimp,phimm)Sx(I,J,T)/Δx

Gy(I,J,T)

5   =Min(phipp,phipm,phimp,phimm)Sy(I,J,T)/Δy ...(38)

Thereafter, the interpolation function

H(I,J,T)(x,y) within the grid (I,J) is defined as:

H(I,J,T)(x,y)=F(I,J,T)+Gx(I,J,T)x+Gy(I,J,T)y ...(39)

The interpolation method according to the present

10   embodiment can be summarized as follows:

A cell of interest is set to a cell A, the cell A

having a first side extending in an x direction, a

second side extending in the x direction and being

opposite to the first side, a third side extending in a

15   y direction, and a fourth side extending in the y

direction and being opposite to the third side, and

values twice as large as values of one-sided

differences of the above described function F between

the center of the cell A and respective centers of four

20   cells adjacent to the cell A on the first side, the

second side, the third side, and the fourth side are

defined as a first-sided difference (DFxmin), a second-

sided difference (DFxmax), a third-sided difference

(DFymin), and a fourth-sided difference (DFymax),

25   respectively. If the first-sided difference and the

second-sided difference have different signs, an x-

direction difference is set to zero, whereas if the

first-sided difference and the second-sided difference
have the same sign, the x-direction difference is set
to the smaller one of the absolute values of the first-
sided difference and the second-sided difference. If
5   the third-sided difference and the fourth-sided
difference have different signs, a y-direction
difference is set to zero, while if the third-sided
difference and the fourth-sided difference have the
same sign, the y-direction difference is set to the
10   smaller one of the absolute values of the third-sided
difference and the fourth-sided difference. There is
formed an interpolation function candidate which has
slopes defined by the x-direction difference and the y-
direction difference, respectively, and has a value F0
15   of the above-mentioned function F at the center of the
cell A, and this candidate is set to a plane B. If a
value of the plane B at a first grid point at a
location of intersection of the first side and the
third side of the cell A is larger than the value of
20   the center of the cell A, the plane B is modified by
multiplying the x-direction difference and the y-
direction difference by a largest constant not more
than 1 such that the value of the plane B at the first
grid point does not exceed any of the values of the
25   function F at respective centers of three cells having
the first grid point in common except for the cell A.
If the value of the plane B at the first grid point is

smaller than the value of the center of the cell A, the

plane B is modified by multiplying the x-direction

difference and the y-direction difference by a largest

constant not more than 1 such that the value of the

5   plane B at the first grid point does not fall below any

of the values of the function F at the respective

centers of the three cells having the grid point in

common except for the cell A. On the plane B thus

obtained, the same operation as carried out as to the

10   first grid point is carried out as to a second grid

point at a location of intersection of the first side

and the fourth side of the cell A, a third grid point

at a location of intersection of the second side and

the third side, and a fourth grid point at a location

15   of intersection of the second side and the fourth side,

to thereby change the slope of the plane B, and the

resulting plane is defined as the interpolation

function H within the cell A. This algorithm is

equivalent to the algorithm described hereinbefore.

20   FIG. 5 is a flowchart showing a process of

advection calculation using the above described

interpolation method according to the second embodiment.

        First, in a step S11, initialization is performed,

e.g. by generating a grid, determining time step

25   intervals, and then initializing the initial shape of a

function $F(\cdot,\cdot,1)$.

        In the following step S12, the computation of the

above-described equation $Q(\cdot,\cdot,J)$ (see the equations (28) to (31)) is performed. At this time, as shown in the above equations (28) and (29), the interpolation function H according to the present embodiment is used.

5　Then, in a step S13, based on data obtained by the equation $Q(\cdot,\cdot,J)$, the computation of the function $F(\cdot,\cdot,J+1)$ is carried out (see the above equation (32)).

Thereafter, time is updated in a step S14. More specifically, the problem being solved now is a time

10　evolution equation expressed by the above equation (26), and according to the conventional practice of numerical solution of an ordinary time evolution equation, the value $F(\cdot,\cdot,J)$ of the function F at time $T=\Delta tJ$ is determined from the value $F(\cdot,\cdot,J-1)$, and then time is

15　changed from $T=\Delta t*(J-1)$ to $T=\Delta t*J$.

Then, in a step S15, it is determined whether or not the present step is a final step, and if the present step is not a final step, the process returns to the step S12, whereby the present computation is

20　performed until it is determined that the present step is a final step.

The use of the interpolation method according to the present embodiment provides two separate regions, one defined by a function with a slope of zero, and the

25　other defined by a function with a sharp slope, which makes it possible to obtain very good results as a function describing the shape of a two-phase

incompressible fluid. FIGS. 6A and 6B show results of the computation. The present computation is carried out under the same conditions corresponding to those of the conventional computations results of which are

5   shown in FIGS. 10A and 10B, and the shape corresponding to the initial value and that after 1000 steps are depicted in FIGS. 10A and 10B, respectively. It will be understood that the result of advection calculation carried out by the interpolation method according to

10   the present embodiment shown in FIG. 6B maintains a very sharp rectangular shape compared with those shown in FIG. 10B.

The computer for executing the advection calculation using the interpolation function according

15   to the present embodiment is identical to those shown in FIGS. 3 and 4.

Next, a description will be given of an interpolation method according to a third embodiment of the present invention. In the third embodiment, the

20   interpolation algorithm for a two-dimensional grid, described hereinabove as to the second embodiment is extended to one for a three-dimensional grid.

The equation dealt with in the present embodiment is a partial differential equation in a three-

25   dimensional space, defined by:

$$\partial F(x,y,z,t) / \partial t = \partial Cx(x,y,z,t) F(x,y,z,t) / \partial x$$

$$+ \partial Cy(x,y,z,t) F(x,y,z,t) / \partial y$$

$$+\partial Cz(x,y,z,t)F(x,y,z,t)/\partial z \quad …(40)$$

The problem of converting this function to a difference equation for computation will now be considered. Suppose a function F(X,Y,Z,T) as a discretized form of F(x,y,z,t). The arguments X, Y, Z in the function F cooperatively represent a discrete spacial grid point and the argument T in the same represents a discrete time.

First, it is assumed that an interpolation function H[I,J,K,T](x,y,z) according to the present embodiment has already been obtained as the following piecewise linear function:

$$H[I,J,K,T](x,y,z)=F(I,J,K,T)+Gx(I,J,K,T)x$$

$$+Gy(I,J,K,T)y+Gz(I,J,K,T)z \quad …(41)$$

With this assumption, a description will be given of a method of numerically solving the above equation (40) by finding a solution thereto by finite difference approximation. A method of determining coefficients Gx (I,J,K,T), Gy(I,J,K,T), and Gz(I,J,K,T) of the interpolation function H[I,J,K,T](x,y,z) of the present embodiment will be described hereinafter. As described later, the method of determining the coefficients Gx(I,J,K,T), Gy(I,J,K,T), and Gz(I,J,K,T) characterizes the interpolation algorithm according to the present embodiment. It is essential to the present embodiment that the coefficients Gx(I,J,K,T), Gy(I,J,K,T), and Gz(I,J,K,T) are determined by using neighboring

information of geometric properties of the grid.

In short, when there are provided a three-dimensional structured grid formed on a three-dimensional real region, and a function $F(X,Y,Z,T)$ on

5   the three-dimensional structured grid, the function being defined through definition of a value thereof at the center of each cell within the three-dimensional structured grid at a given time T, the function $F(X,Y,Z,T)$ is defined as an interpolation function

10    $H[I,J,K, T](x,y,z)$ on the three-dimensional real region by extending and interpolating the function $F(X,Y,Z,T)$ through utilization of the function $F(X,Y,Z,T)$ and the geometry of the grid.

If the function $Cx(I+1/2,J, K,T)$ is positive,

15   there is given the following definition:

$Qx(I+1/2,J,K,T):=Cx(I+1/2,J,K,T)H[I,J,K,T]$

$(\Delta x/2-Cx(I,J,K,T)\Delta t/2,-Cy(I,J,K,T)\Delta t/2,$

$-Cz(I,J,K,T)\Delta t/2)$ …(42)

If the function $Cx(I+1/2,J,K,T)$ is not positive,

20   there is given the following definition:

$Qx(I+1/2,J,K,T):=Cx(I+1/2,J,K,T)H[I+1,J,K,T]$

$(-\Delta x/2+Cx(I,J,K,T)\Delta t/2,-Cy(I,J,K,T)\Delta t/2,$

$-Cz(I,J,K,T)\Delta t/2)$ …(43)

If the function $Cy(I,J+1/2,K,T)$ is positive, there

25   is given the following definition:

$Qy(I,J+1/2,K,T):=Cy(I,J+1/2,K,T)H[I,J,K,T]$

$(-Cx(I,J,K,T)\Delta t/2,\Delta y/2-Cy(I,J,K,T)\Delta t/2,$

$-Cz(I,J,K,T)\Delta t/2)$ ...(44)

If the function $Cy(I,J+1/2,K,T)$ is not positive, there is given the following definition:

$Qy(I,J+1/2,K,T):=Cy(I,J+1/2,K,T)H[I,J+1,K,T]$

$(-Cx(I,J,K,T)\Delta t/2,-\Delta y/2-Cy(I,J,K,T)\Delta t/2,$

$-Cz(I,J,K,T)\Delta t/2)$ ...(45)

Further, if the function $Cz(I,J,K+1/2,T)$ is not positive, there is given the following definition:

$Qz(I,J,K+1/2,T):=Cz(I,J,K+1/2,T)H[I,J,K,T]$

$(-Cx(I,J,K,T)\Delta t/2,-Cy(I,J,K,T)\Delta t/2,$

$\Delta z/2-Cz(I,J,K,T)\Delta t/2)$ ...(46)

If the function $Cy(I,J,K+1/2,T)$ is not positive, there is given the following definition:

$Qz(I,J,K+1/2,T):=Cz(I,J,K+1/2,T)H[I,J,K+1,T]$

$(-Cx(I,J,K,T)\Delta t/2,-Cy(I,J,K,T)\Delta t/2,$

$-\Delta z/2-Cz(I,J,K,T)\Delta t/2)$ ...(47)

From the above definitions, there is obtained the following equation:

$F(I,J,K,T+1)=F(I,J,K,T)+\Delta t(Qx(I+1/2,J,K,T)$

$-Qx(I-1/2,J,K,T))/\Delta x+\Delta t(Qy(I,J+1/2,K,T)$

$-Qy(I,J-1/2,K,T))/\Delta y+\Delta t(Qz(I,J,K+1/2,T)$

$-Qz(I,J,K-1/2,T))/\Delta z$ ...(48)

On the other hand, a slope function $G(I,J,K,T)$ representative of the slope of the piecewise linear function is defined in the following manner:

First, there are introduced the following definitions:

$$Sxp(I,J,K,T):=F(I+1,J,K,T)-F(I,J,K,T)$$

$$Sxm(I,J,K,T):=F(I,J,K,T)-F(I-1,J,K,T)$$

$$Syp(I,J,K,T):=F(I,J+1,K,T)-F(I,J,K,T)$$

$$Sym(I,J,K,T):=F(I,J,K,T)-F(I,J-1,K,T)$$

$$Szp(I,J,K,T):=F(I,J,K+1,T)-F(I,J,K,T)$$

$$Szm(I,J,K,T):=F(I,J,K,T)-F(I,J,K-1,T) \quad …(49)$$

Further, there are introduced limiter functions in one dimension for the present embodiment, defined by:

$$Sx(I,J,K)=(sgn(Sxp(I,J,K,T))+sgn(Sxm(I,J,K,T)))$$

$$Min(|Sxp(I,J,K,T)|,|Sxm(I,J,K,T)|)$$

$$Sy(I,J,K)=(sgn(Syp(I,J,K,T))+sgn(Sym(I,J,K,T)))$$

$$Min(|Syp(I,J,K,T)|,|Sym(I,J,K,T)|)$$

$$Sz(I,J,K)=(sgn(Szp(I,J,K,T))+sgn(Szm(I,J,K,T)))$$

$$Min(|Szp(I,J,K,T)|,|Szm(I,J,K,T)|) \quad …(50)$$

Then, there are provided the following definitions:

$$Vpppmin=Min(F(I,J+1,K,T)-F(I,J,K,T),F(I+1,J+1,K,T)$$

$$-F(I,J,K,T),-\varepsilon,F(I+1,J,K,T)-F(I,J,K,T),$$

$$F(I,J+1,K+1,T)-F(I,J,K,T),F(I+1,J+1,K+1,T)$$

$$-F(I,J,K,T),F(I,J,K+1,T)-F(I,J,K,T),$$

$$F(I+1,J,K+1,T)-F(I,J,K,T))$$

$$Vmppmin=Min(F(I-1,J+1,K,T)-(I,J,K,T),F(I,J+1,K,T)$$

$$-F(I,J,K,T),F(I-1,J,K,T)-F(I,J,K,T),-\varepsilon,$$

$$F(I-1,J+1,K+1,T)-F(I,J,K,T),F(I,J+1,K+1,T)$$

$$-F(I,J,K,T),F(I-1,J,K+1,T)-F(I,J,K,T),$$

$$F(I,J,K+1,T)-F(I,J,K,T))$$

$$Vpmpmin=Min(-\varepsilon,F(I+1,J,K,T)-F(I,J,K,T),$$

$F(I,J-1,K,T)-F(I,J,K,T),F(I+1,J-1,K,T)-F(I,J,K,T),$

$F(I,J,K+1,T)-F(I,J,K,T),F(I+1,J,K+1,T)-F(I,J,K,T),$

$F(I,J-1,K+1,T)-F(I,J,K,T),F(I+1,J-1,K+1,T)$

$-F(I,J,K,T))$

5 $Vmmpmin=Min(F(I-1,J,K,T)-F(I,J,K,T),-\varepsilon,$

$F(I-1,J-1,K,T)-F(I,J,K,T),F(I,J-1,K,T)-F(I,J,K,T),$

$F(I-1,J,K+1,T)-F(I,J,K,T),F(I,J,K+1,T)-F(I,J,K,T),$

$F(I-1,J-1,K+1,T)-F(I,J,K,T),F(I,J-1,K+1,T)$

$-F(I,J,K,T))$

10 $Vppmmin=Min(F(I,J+1,K,T)-(I,J,K,T),F(I+1,J+1,K,T)$

$-F(I,J,K,T),-\varepsilon,F(I+1,J,K,T)-F(I,J,K,T),$

$F(I,J+1,K-1,T)-F(I,J,K,T),F(I+1,J+1,K-1,T)$

$-F(I,J,K,T),F(I,J,K-1,T)-F(I,J,K,T),$

$F(I+1,J,K-1,T)-F(I,J,K,T))$

15 $Vmpmmin=Min(F(I-1,J+1,K,T)-(I,J,K,T),F(I,J+1,K,T)$

$-F(I,J,K,T),F(I-1,J,K,T)-F(I,J,K,T),-\varepsilon,$

$F(I-1,J+1,K-1,T)-F(I,J,K,T),F(I,J+1,K-1,T)$

$-F(I,J,K,T),F(I-1,J,K-1,T)-F(I,J,K,T),$

$F(I,J,K-1,T)-F(I,J,K,T))$

20 $Vpmmmin=Min(-\varepsilon,F(I+1,J,K,T)-F(I,J,K,T),$

$F(I,J-1,K,T)-F(I,J,K,T),F(I+1,J-1,K,T)-F(I,J,K,T),$

$F(I,J,K-1,T)-F(I,J,K,T),F(I+1,J,K-1,T)-F(I,J,K,T),$

$F(I,J-1,K-1,T)-F(I,J,K,T),F(I+1,J-1,K-1,T)$

$-F(I,J,K,T))$

25 $Vmmmmin=Min(F(I-1,J,K,T)-F(I,J,K,T),-\varepsilon,$

$F(I-1,J-1,K,T)-F(I,J,K,T),F(I,J-1,K,T)-F(I,J,K,T),$

$F(I-1,J,K-1,T)-F(I,J,K,T),F(I,J,K-1,T)-F(I,J,K,T),$

$F(I-1,J-1,K-1,T)-F(I,J,K,T),F(I,J-1,K-1,T)$

$-F(I,J,K,T))$ ...(51)

$Vpppmax=Max(F(I,J+1,K,T)-(I,J,K,T),F(I+1,J+1,K,T)$

$-F(I,J,K,T),-\varepsilon,F(I+1,J,K,T)-F(I,J,K,T),$

$F(I,J+1,K+1,T)-F(I,J,K,T),F(I+1,J+1,K+1,T)$

$-F(I,J,K,T),F(I,J,K+1,T)-F(I,J,K,T),$

$F(I+1,J,K+1,T)-F(I,J,K,T))$

$Vmppmax=Max(F(I-1,J+1,K,T)-(I,J,K,T),F(I,J+1,K,T)$

$-F(I,J,K,T),F(I-1,J,K,T)-F(I,J,K,T),-\varepsilon,$

$F(I-1,J+1,K+1,T)-F(I,J,K,T),F(I,J+1,K+1,T)$

$-F(I,J,K,T),F(I-1,J,K+1,T)-F(I,J,K,T),$

$F(I,J,K+1,T)-F(I,J,K,T))$

$Vpmpmax=Max(-\varepsilon,F(I+1,J,K,T)-F(I,J,K,T),$

$F(I,J-1,K,T)-F(I,J,K,T),F(I+1,J-1,K,T)-F(I,J,K,T),$

$F(I,J,K+1,T)-F(I,J,K,T),F(I+1,J,K+1,T)-F(I,J,K,T),$

$F(I,J-1,K+1,T)-F(I,J,K,T),F(I+1,J-1,K+1,T)$

$-F(I,J,K,T))$

$Vmmpmax=Max(F(I-1,J,K,T)-F(I,J,K,T),-\varepsilon,$

$F(I-1,J-1,K,T)-F(I,J,K,T),F(I,J-1,K,T)-F(I,J,K,T),$

$F(I-1,J,K+1,T)-F(I,J,K,T),F(I,J,K+1,T)-F(I,J,K,T),$

$F(I-1,J-1,K+1,T)-F(I,J,K,T),F(I,J-1,K+1,T)$

$-F(I,J,K,T))$

$Vppmmax=Max(F(I,J+1,K,T)-(I,J,K,T),F(I+1,J+1,K,T)$

$-F(I,J,K,T),-\varepsilon,F(I+1,J,K,T)-F(I,J,K,T),$

$F(I,J+1,K-1,T)-F(I,J,K,T),F(I+1,J+1,K-1,T)$

$-F(I,J,K,T),F(I,J,K-1,T)-F(I,J,K,T),$

$F(I+1,J,K-1,T)-F(I,J,K,T))$

Vmpmmax=Max(F(I-1,J+1,K,T)-(I,J,K,T),F(I,J+1,K,T)

-F(I,J,K,T),F(I-1,J,K,T)-F(I,J,K,T),- ε ,

F(I-1,J+1,K-1,T)-F(I,J,K,T),F(I,J+1,K-1,T)

-F(I,J,K,T),F(I-1,J,K-1,T)-F(I,J,K,T),

5   F(I,J,K-1,T)-F(I,J,K,T))

Vpmmmax=Max(- ε ,F(I+1,J,K,T)-F(I,J,K,T),

F(I,J-1,K,T)-F(I,J,K,T),F(I+1,J-1,K,T)-F(I,J,K,T),

F(I,J,K-1,T)-F(I,J,K,T),F(I+1,J,K-1,T)-F(I,J,K,T),

F(I,J-1,K-1,T)-F(I,J,K,T),F(I+1,J-1,K-1,T)

10   -F(I,J,K,T))

Vmmmmax=Max(F(I-1,J,K,T)-F(I,J,K,T),- ε ,

F(I-1,J-1,K,T)-F(I,J,K,T),F(I,J-1,K,T)-F(I,J,K,T),

F(I-1,J,K-1,T)-F(I,J,K,T),F(I,J,K-1,T)-F(I,J,K,T),

F(I-1,J-1,K-1,T)-F(I,J,K,T),F(I,J-1,K-1,T)

15   -F(I,J,K,T)) …(52)

There are also provided the following definitions:

hppp(I,J,K,T)

=Sx(I,J,K,T)/2+Sx(I,J,K,T)/2+Sz(I,J,K,T)/2

hmpp(I,J,K,T)

20   =-Sx(I,J,K,T)/2+Sx(I,J,K,T)/2+Sz(I,J,K,T)/2

hpmp(I,J,K,T)

=Sx(I,J,K,T)/2-Sx(I,J,K,T)/2+Sz(I,J,K,T)/2

hmmp(I,J,K,T)

=-Sx(I,J,K,T)/2-Sx(I,J,K,T)/2+Sz(I,J,K,T)/2

25   hppm(I,J,K,T)

=Sx(I,J,K,T)/2+Sx(I,J,K,T)/2-Sz(I,J,K,T)/2

hmpm(I,J,K,T)

$$=-Sx(I,J,K,T)/2+Sx(I,J,K,T)/2-Sz(I,J,K,T)/2$$

hpmm(I,J,K,T)

$$=Sx(I,J,K,T)/2-Sx(I,J,K,T)/2-Sz(I,J,K,T)/2$$

hmmm(I,J,K,T)

5      $$=-Sx(I,J,K,T)/2-Sx(I,J,K,T)/2-Sz(I,J,K,T)/2 \quad …(53)$$

Further, there is performed the following calculation:

Assuming, for the time being, that an equation phippp=phimpp=phipmp=phimmp=phippm=phimpm=phipmm=phimmm

10   =1 holds,

if hppp>Vpppmax holds, an equation of phippp=Vpppmax/hppp is newly defined, while if hppp<Vpppmin holds, an equation of phippp=Vpppmin/hppp is newly defined;

15   Similarly, if hpmp>Vpmpmax holds, an equation of phipmp=Vpmpmax/hpmp is newly defined, while if hpmp<Vpmpmin holds, an equation of phipmp=Vpmmin/hpmp is newly defined;

Also, if hmpp>Vmppmax holds, an equation of

20   phimpp=Vmppmax/hmpp is newly defined, while if hmpp<Vmppmin holds, an equation of phimpp=Vmppmin/hmpp is newly defined;

If hmmp>Vmmpmax holds, an equation of phimmp=Vmmpmax/hmmp is newly defined, while if

25   hmmp<Vmmpmin holds, an equation of phimmp=Vmmpmin/hmmp is newly defined;

If hppm>Vppmmax holds, an equation of

phippm=Vppmmax/hppm is newly defined, while if

hppm<Vppmmin holds, an equation of phippm=Vppmmin/hppm

is newly defined;

If hpmm>Vpmmmax holds, an equation of

5   phipmm=Vpmmmax/hpmm is newly defined, while if

hpmm<Vpmmmin holds, an equation of phipmm=Vpmmmin/hpmm

is newly defined;

If hmpm>Vmpmmax holds, an equation of

phimpm=Vmpmmax/hmpm is newly defined, while if

10   hmpm<Vmpmmin holds, an equation of phimpm=Vmpmmin/hmpm

is newly defined; and

If hmmm>Vmmmmax holds, an equation of

phimmm=Vmmmmax/hmmp is newly defined, while if

hmmm<Vmmmmin holds, an equation of phimmm=Vmmmmin/hmmm

15   is newly defined.

By using the thus obtained values phippp, phimpp,

phipmp, phimmp, phippm, phimpm, phipmm, phimmm, the

above-mentioned coefficients Gx(I,J,K,T), Gy(I,J,K,T),

and Gz(I,J,K,T) are determined by the following

20   equations:

Gx(I,J,K,T)=Min(phippp,phimpp,phipmp,phimmp,phippm,

phimpm,phipmm,phimmm)Sx(I,J,K,T)/Δx

Gy(I,J,K,T)=Min(phippp,phimpp,phipmp,phimmp,phippm,

phimpm,phipmm,phimmm)Sy(I,J,K,T)/Δy

25   Gz(I,J,K,T)=Min(phippp,phimpp,phipmp,phimmp,phippm

phimpm,phipmm,phimmm)Sz(I,J,K,T)/Δz

Thereafter, the interpolation function

H(I,J,K,T)(x,y,z) within the grid (I,J,K) is defined
as:

$$H[I,J,K,T](x,y,z)=F(I,J,K,T)+Gx(I,J,K,T)x$$

$$+Gy(I,J,K,T)y+Gz(I,J,K,T)z \quad …(54)$$

5   The interpolation method according to the present
embodiment can be summarized as follows:

A cell of interest is set to a cell A, the cell A
having a first side (face, in this three-dimensional
grid) extending in an x direction, a second side

10 extending in the x direction and being opposite to the
first side, a third side extending in a y direction, a
fourth side extending in the y direction and being
opposite to the third side, a fifth side in a z
direction, and a sixth side extending in the z

15 direction and being opposite to the fifth side, and
values twice as large as values of one-sided
differences of the above described function F between
the center of the cell A and respective centers of six
cells adjacent to the cell A on the first side, the

20 second side, the third side, the fourth side, the fifth
side, and the sixth side are defined as a first-sided
difference (DFxmin), a second-sided difference (DFxmax),
a third-sided difference (DFymin), a fourth-sided
difference (DFymax), a fifth-sided difference (DFzmax),

25 and a sixth-sided difference (DFzmin), respectively.
If the first-sided difference and the second-sided
difference have different signs, an x-direction

difference is set to zero, whereas if the first-sided difference and the second-sided difference have the same sign, the x-direction difference is set to the smaller one of the absolute values of the first-sided

5   difference and the second-sided difference. If the third-sided difference and the fourth-sided difference have different signs, a y-direction difference is set to zero, while if the third-sided difference and the fourth-sided difference have the same sign, the y-

10 direction difference is set to the smaller one of the absolute values of the third-sided difference and the fourth-sided difference. If the fifth-sided difference and the sixth-sided difference have different signs, a z-direction difference is set to zero, while if the

15 fifth-sided difference and the sixth-sided difference have the same sign, the z-direction difference is set to the smaller one of absolute values of the fifth-sided difference and the sixth-sided difference. There is formed an interpolation function candidate which has

20 slopes defined by the x-direction difference, the y-direction difference, and the z-direction difference, respectively, and has a value F0 of the above-mentioned function F at the center of the cell A, and this candidate is set to a plane B. If a value of the plane

25 B at a first grid point at a location of intersection of the first side, the third side, and the fifth side of the cell A is larger than the value of the center of

the cell A, the plane B is modified by multiplying the
x-direction difference, the y-direction difference, and
the z-direction difference by a largest constant not
more than 1 such that the value of the plane B at the

5   first grid point does not exceed any of the values of
the function F at respective centers of seven cells
having the first grid point in common except for the
cell A. If the value of the plane B at the first grid
point is smaller than the value of the center of the

10   cell A, the plane B is modified by multiplying the x-
direction difference, the y-direction difference, and
the z-direction difference by a largest constant not
more than 1 such that the value of the plane B at the
first grid point does not fall below any of the values

15   of the function f at the respective centers of the
seven cells having the grid point in common except for
the cell A.

On the plane B as the candidate interpolation
function thus obtained, the same operation as carried

20   out as to the first grid point is carried out as to a
second grid point at a location of intersection of the
first side, the third side, and the sixth side, a third
grid point at a location of intersection of the first
side, the fourth side, and the fifth side, a fourth

25   grid point at a location of intersection of the first
side, the fourth side, and the sixth side, a fifth grid
point at a location of intersection of the second side,

the third side, and the fifth side, and a sixth grid

point at a location of intersection of the second side,

the third side, and the sixth side, a seventh grid

point at a location of intersection of the second side,

5   the fourth side, and the fifth side, and an eighth grid

point at a location of intersection of the second side,

the fourth side, and the sixth side, to thereby change

the slope of the plane B, and the resulting plane is

defined as the interpolation function H within the cell

10   A. This algorithm is equivalent to the algorithm

described hereinbefore.

A flowchart of an advection calculation using the

interpolation method of the present embodiment is the

same as shown in FIG. 5. Further, the interpolation

15   method according to the third embodiment can be

realized by the same computer described hereinbefore

with reference to FIGS. 3 and 4. In the present

embodiment as well, it is possible to carry out the

advection calculation while maintaining a sharp shape

20   of the function as is the case with the interpolation

method for a two-dimensional space according to the

above described second embodiment.

If the interpolation methods according to the

second and third embodiments are used, it is possible

25   to form a function in a two-dimensional or three-

dimensional space based on values given to grid points

of the space although the values include

discontinuities. By virtue of this feature, even when the interpolation method according to the present invention is applied to other uses than the above described embodiments, e.g. reproduction of image data,

5 detailing data of a digital two-dimensional or three-dimensional object, it is possible to express a sharp shape in the same manner.

Furthermore, the present invention is not limited to the apparatuses according to the above described

10 embodiments but may be either applied to a system composed of a plurality of apparatuses or to a single apparatus.

It is to be understood that the object of the present invention may be accomplished by supplying a

15 system or an apparatus with a storage medium in which a program code of software which realizes the functions of any of the above described embodiments, and causing a computer (CPU or MPU) of the system or apparatus to read out and execute the program code stored in the

20 storage medium.

In this case, the program code itself read from the storage medium realizes the functions of any of the above described embodiments, and hence the storage medium on which the program code is stored constitutes

25 the present invention.

Examples of the storage medium for supplying the program code include a floppy (registered trademark)

disk, a hard disk, an optical disk, a magneto optical disk, a CD-ROM, a CD-R, a CD-RW, a DVD-ROM, a DVD-RAM, a DVD-RW, a DVD+RW, a magnetic tape, a nonvolatile memory card, and a ROM. Downloading via a network can

5 also be utilized.

Further, it is to be understood that the functions of any of the above described embodiments may be accomplished not only by executing a program code read out by a computer, but also by causing an OS (operating

10 system) or the like which operates on the computer to perform a part or all of the actual operations based on instructions of the program code.

Further, it is to be understood that the functions of any of the above described embodiments may be

15 accomplished by writing a program code read out from the storage medium into a memory provided on an expansion board inserted into a computer or in an expansion unit connected to the computer and then causing a CPU or the like provided in the expansion

20 board or the expansion unit to perform a part or all of the actual operations based on instructions of the program code.